Sequences & series

Sequence : List of number in a Specific order

e.g : 0, 1, 5, 100, 1000

☆ Arithmetic progression

Def^n : AP is a sequence in which each term increase or decrease by a fixed number

fixed number is called common difference (d)

☆ General term of AP

$$a, \; a+d, \; a+2d, \; a+3d \; \ldots$$

$$a_1 \quad a_2 \quad a_3 \quad a_4$$

$$\boxed{a_n = a + (n-1)d}$$

☆ Series

Series is representation of sum of term of AP

2, 5, 8, 11

Series = 2 + 5 + 8 + 11

☆ Sum of first n terms of AP (S_n)

$$\boxed{S_n = \frac{n}{2}[2a+(n-1)d]}$$

☆ Sum of last term and first term

$$\boxed{S_n = \frac{n}{2}[a+l]}$$

☆ Some Imp Result

☆ Sum of first n natural number

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

Sum of first n odd natural number

$1+3+5+\dots+(2n-1) = n^2 = (\text{number of terms})$

☆ $\boxed{a_n = S_n - S_{n-1}}$ IMP

☆ Properties of AP

1) Common difference can be positive, negative or zero

2) If 3 terms (a, b, c) in AP

$$\boxed{2b = a+c}$$

3) Considering the terms of AP

3 terms in AP = $a-d, a, a+d$

4 terms in AP = $a-3d, a-d, a+d, a+3d$

5 term in AP = $a-2d, a-d, a, a+d, a+2d$

4) Sum of terms of the AP equidistance form beginning & end is constant and it is equal to sum of first & last term

$a, a+d, a+2d, a+3d, a+4d, a+5d$

$= a+d+a+4d$

$= \boxed{2a+5d}$

5) ~~Energy~~ Every term of AP is average of equidistance term from it

$a + a+d + a+2d + a+3d + a+4d$

$= \frac{a+d+a+3d}{2} = \frac{2a+4d}{2}$

$= a+2d$

$$\boxed{a_n = \frac{a_{n-k} + a_{n+k}}{2}}$$

6) If each term of AP increased, decreased multiplied or devided by same number then new sequence will also be in AP

e.g $2, 4, 6, 8, \dots, 12$

$+1$ $3, 5, 7, 9, \dots, 13$

-2 $0, 2, 4, 6, \dots, 10$

$\times 2$ $4, 8, 12, 16, \dots, 24$

$\div 2$ $1, 2, 3, 4, \dots, 6$

☆ Geometric progression (GP)

Defn : GP is sequence in which first term is non-zero and each of the succeding term is obtained by multiplying a constant to previous term

e.g devided by 2

$3, 6, 12, 24$

first term $= a$

Common ratio $= r = \frac{a_2}{a_1} = \frac{a_n}{a_{n-1}}$

☆ General term of GP

a, ar, ar^2, ar^3

$$\boxed{\text{General term} \quad a_n = ar^{n-1}}$$

2. Powers of 2

$2^1 = 2$	$3^1 = 3$	$4^1 = 4$
$2^2 = 4$	$3^2 = 9$	$4^2 = 16$
$2^3 = 8$	$3^3 = 27$	$4^3 = 48$
$2^4 = 16$	$3^4 = 81$	$4^4 = 144$
$2^5 = 32$	$3^5 = 243$	$4^5 = 432$
$2^6 = 64$	$3^6 = 729$	$4^6 = 1296$
$2^7 = 128$	$3^7 = 2187$	$4^7 = 3888$
$2^8 = 256$	$3^8 = 6561$	$4^9 = 11664$
$2^9 = 512$	$3^9 = 19683$	$4^9 = 34992$
$2^{10} = 1024$	$3^{10} = 59049$	$4^{10} = 104979$
$2^{11} = 2048$	$3^{11} = 177147$	$4^{11} =$
$2^{12} = 4096$	$3^{12} = 531441$	$4^{12} =$
$2^{13} = 8192$	$3^{13} = 1594323$	$4^{13} =$
$2^{14} = 16384$	$3^{14} = 4782969$	$4^{14} =$
$2^{15} = 32768$	$3^{15} = 14348903$	$4^{15} =$
$2^{16} = 65536$	$3^{16} = 43046721$	$4^{16} =$
$2^{17} = 131072$	$3^{17} = 129140163$	$4^{17} =$
$2^{18} = 262144$	$3^{18} = 38420489$	$4^{18} =$
$2^{19} = 524288$	$3^{19} = 1162261467$	$4^{19} =$
$2^{20} = 1048576$	$3^{20} = 3486784401$	$4^{20} =$

$5^1 = 5$	$6^1 = 6$	$7^1 = 7$	$8^1 =$
$5^2 = 25$	$6^2 = 36$	$7^2 = 49$	$8^2 =$
$5^3 = \cancel{5}125$	$6^3 = 216$	$7^3 = 343$	$8^3 =$
$5^4 = 625$	$6^4 = 1296$	$7^4 = 2401$	$8^4 =$
$5^5 = 3125$	$6^5 = 7776$	$7^5 = 16807$	$8^5 =$
$5^6 = 15625$	$6^6 = 46656$	$7^6 = 117649$	$8^6 =$
$5^7 = 78125$	$6^7 = 279936$	$7^7 = 823543$	$8^7 =$
$5^8 = 390625$	$6^8 = 1679616$	$7^8 = 5764801$	$8^8 =$
$5^9 = 1953125$	$6^9 = 10077696$	$7^9 = 4035360$	$8^9 =$
$5^{10} = 9765625$	$6^{10} = 60466176$	$7^{10} =$	$8^{10} =$

$9^1 =$	$10^1 =$	$11^1 =$	$12^1 =$
$9^2 =$	$10^2 =$	$11^2 =$	$12^2 =$
$9^3 =$	$10^3 =$	$11^3 =$	$12^3 =$
$9^4 =$	$10^4 =$	$11^4 =$	$12^4 =$
$9^5 =$	$10^5 =$	$11^5 =$	$12^5 =$
$9^6 =$	$10^6 =$	$11^6 =$	$12^6 =$
$9^7 =$	$10^7 =$	$11^7 =$	$12^7 =$
$9^8 =$	$10^8 =$	$11^8 =$	$12^8 =$
$9^9 =$	$10^9 =$	$11^9 =$	$12^9 =$
$9^{10} =$	$10^{10} =$	$11^{10} =$	$12^{10} =$

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★ Sum of GP

1) Sum of finite term of GP

$$S_n = a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

$$= \boxed{\frac{a(1-r^n)}{1-r}}$$

2) Sum of infinite term of GP

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \dots \infty$

Note

$n \to \infty \quad r^n = 0$

$|r| < 1$

$\to -1 \dots 1$

$$\boxed{S_n = \frac{a}{1-r} \qquad |r| < 1}$$

3) Considering number in GP

3 terms $\frac{a}{r}$, a, ar ⇒ common ratio → r

4 terms

$\frac{a}{r^3}$, $\frac{a}{r}$, ar, ar^3 ⇒ Common ratio $= r^2$

4) Product of equidistance terms from begening and end is constant

$a, ar, ar^2, ar^3, ar^5, ar^6, ar^2$

first ... last

Value $= a^2r^7$

6) If terms of GP multiplied or devided or power raised by some quantity then resultant sequence is also a GP

a, ar, ar^2, ar^3, ar^4

* multiply by k

→ $ka, kar, kar^2, kar^3, kar^4$ → Common ratio $= r$

* devided by k

$\frac{a}{k}, \frac{ar}{k}, \frac{ar^2}{k}, \frac{ar^3}{k}, \frac{ar^2}{k}$

Common ratio $= r$

* power raised to k

$a^k, ar^k, (ar^2)^k, (ar^3)^k, (ar^4)^k$

Common ratio $= (r^k)$

6) If $a_1, a_2, a_3 \ldots a_n$ & $b_1, b_2, b_3 \ldots b_n$ we two GPs then $a_1 \cdot b_1, a_2 \cdot b_2$, $a_3 \cdot b_3 \ldots a_n b_n$ is also in GP

e.g $a, ar_1, ar_1^2, \ldots ar_1^{n-1}$

$b, br_2, br_2^2 \ldots br_2^{n-1}$

$\rightarrow ab, abr_1r_2, abr_1^2r_2^2 \ldots \ldots abr_1^{n-1} \cdot r_2^{n-1}$

Common ratio $= r_1 \cdot r_2$

7) If $a, ar, ar^2, ar^3 \ldots$ is G.P then $\log a, \log(ar), \log(ar^2)$ in AP

$\Rightarrow \log a, \log a + \log r, \log a + 2\log r, \log a + 3\log r$

$\boxed{\text{first term} = \log a}$

$\boxed{\text{Common distance} = \log r}$